**Fama French Model or Three Factor Model**

Though CAPM contends that only driver or only factor of equity return is the market factor but after 1980s, substantial empirical evidence showed that market factor alone failed to explain the return. In the US and other equity market, evidence suggests that Small cap stocks or value stocks generate higher return over the long run than the CAPM predicts.

In 1993 researcher Fama and French addressed the perceived weakness of the CAPM in a model with Three Factors, which is known as Fama French Model (FFM) or Three Factor Model (TFM).

The 3 factors of return in the FFM is namely

1. **RMRF or market factor** similar to CAPM, which is measured as return on market value weighted equity index in excess of risk free rate
2. **SMB ( Small Minus Big) or size factor** which is the return to a portfolio of small capitalization stocks less the return to a portfolio of large capitalization stocks
3. **HML(i.e., high minus low) or value factor** which is the return to a portfolio of stocks with high ratios of book-to market values less the return to a portfolio of low book-to-market value stocks

The FFM estimate of required return on equity can be expressed as

**Ri = Rf + β1 mkt factorRMRF + β2 size factorSMB + β3 value factorHML**

The FFM equation says that required return on equity is not just only function of market factor as in CAPM, two additional factors of return - the size factor and the value factor play role in the required return on equity.

Like market risk factor premium, average historical estimate of size factor and value factor premium is positive. This means that investor will require higher required for holding small size company compared to average size company though both small size and average size company have the same market beta. The logic and intuition is that small company is relatively more risky than an average size company and CAPM market beta cannot capture this risk arising from the size factor. Similar to size, required return on high value company (company having high book value to market price ratio) is higher compared to average value company though both high value and average value company have the same market beta

Like CAPM, neutral or average value of beta (B1) for market factor in the FFM is 1 , but neutral value or average value of beta for size and value factor is zero . For a security, the beta value of zero for size and value factor means that the security has no size or value bias. A beta value of positive for size factor means that security is a smaller company relative to average size company, similarly a negative beta value for size factor means that security has relatively higher size than a average size company in the market.

**Illustration of Estimation of Historical Market , Size and Value Factor Premium**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Year****1** | **Market Return (DSEX)** | **Risk Free Rate** | **Market Premium** | **Return on Small cap company** | **Return on Big cap company** | **Size Factor Premium** | **Return on company with High BV to MP ratio****8** | **Return on company with low BV to MP ratio** **9** | **Value factor Premium** |
|  |  |  |  | **6** |  |  |
|  2 | **3** | **4= 3-2** | **5** |   | **7=(6-5)** | **10= (9—8)** |
| **1** | 15.00% | 10% | **5%** | 18.00% | 14.00% | **4.00%** | 16.50% | 14.50% | **2.00%** |
| **2** | 22.00% | 10% | **12%** | 25.00% | 21.00% | **4.00%** | 23.50% | 21.50% | **2.00%** |
| **3** | 13.00% | 10% | **3%** | 15.50% | 12.00% | **3.50%** | 18.50% | 12.50% | **6.00%** |
| **4** | 24.00% | 10% | **14%** | 26.50% | 19.00% | **7.50%** | 29.50% | 23.50% | **6.00%** |
| **5** | 15.00% | 10% | **5%** | 17.50% | 14.00% | **3.50%** | 20.50% | 14.50% | **6.00%** |
| **6** | 26.00% | 10% | **16%** | 28.50% | 25.00% | **3.50%** | 27.50% | 25.50% | **2.00%** |
| **7** | -12.00% | 10% | **-22%** | -9.50% | -13.00% | **3.50%** | -10.50% | -12.50% | **2.00%** |
| **8** | 15.00% | 10% | **5%** | 17.50% | 14.00% | **3.50%** | 16.50% | 14.50% | **2.00%** |
| **9** | 30.00% | 10% | **20%** | 32.50% | 29.00% | **3.50%** | 32.50% | 29.50% | **3.00%** |
| **10** | 27.00% | 10% | **17%** | 29.50% | 26.00% | **3.50%** | 29.50% | 26.50% | **3.00%** |
| **11** | 22.00% | 10% | **12%** | 26.50% | 21.00% | **5.50%** | 23.50% | 21.50% | **2.00%** |
| **12** | 20.00% | 10% | **10%** | 22.50% | 16.00% | **6.50%** | 21.50% | 19.50% | **2.00%** |
| **Average** | **18.08%** | **10%** | **8%** | **20.83%** | **16.50%** | **4.33%** | **20.75%** | **17.58%** | **3.17%** |

The table above shows market risk premium RMRF is 8%, Size factor premium is 4.33% and the value factor premium is 3.17%, so the FFM equation stands at

**Ri = Rf + β1\*8% + β2 4.33% + β3 3.17%**

**S**uppose a share of company X has sensibility to market factor or B1 is  1.1 and sensibility to size factor or B2 is 1.5 and sensibility to value factor is B3 -0.5, so the required return on share X is

Also assume that risk free rate is 6%,

Rx = 6% + 8%\*1.1 + 1.5\* 4.33% + 3.17% \* - (0.5)

Rx= 19.71%

FFM estimates required return to be 19.71% whereas if CAPM is used the required return would have been

Rx = 6% + 8%\*1.1

 = 14.8%

CAPM required return is lower than FFM return because CAPM assumes that investors do not require size factor premium and value factor premium and this factor premium are already in market factor premium or this size factor and value factor premium are due to market inefficiency. But considerable studies and empirical work has shown that small cap company and high value company outperforms big cap and low value company over a long period of time horizon both in US and other developed market.

FFM says in the above example that the required return of 19.71 % comes from three risk sources, market risk , size factor and value factor risk. Moreover the FFM also says that company X is a very small size company as depicted by the size factor beta of 1.5 and size factor contributes to its return of 4.33% \*1.5 = 6.5%. Beta to value factor of -0.5 means that X is a low value company (or growth company) and this value factor contributes negatively to its required return.

With FFM, it is also possible to construct portfolio to the investment objective of portfolio manager.

For example, suppose 3 companies A, B, C are to be combined in a portfolio and investment manager want that portfolio return **to go up by 2**% when market return **goes up by 1%** and portfolio return to **go up by 3%** when market return of small size companies **go up by 1%** and portfolio **return to go up by 2%** when portfolio of value company **goes down by 1%.** Following information is available

|  |
| --- |
| **Factor Risk Premium Estimate**  |
| Market factor premium | 7% |
| Size factor premium | 4% |
| Value factor premium | 3% |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Company**  | **Beta or sensibility to market factor** | **Beta or sensibility to size factor**  | **Beta or sensibility to value factor** | **Expected Return** |
| A |  1.20  |  0.50  |  (0.50) | 15% |
| B |  1.00  |  2.00  |  (1.00) | 20% |
| C |  1.50  |  (1.00) |  2.00  | 18% |

Now if we want to build such portfolio of stocks A, B, C then according to requirement, Portfolio should be following characteristics

Portfolio market beta = 1

Portfolio size beta = 3 and

Portfolio value factor beta = -2

So weight of each of stock in the portfolio should be such that

WA\*1.2 +WB \*1 + WC \*1.5 = 2

WA\*.5 +WB \*2+ WC \*(-1) = 3

WA\*(-.5) +WB \*(-1) + WC \*2 = -2

Now if we solve this simultaneous equation , we can find the weight of A , B and C to construct a portfolio having exposure or beta to market factor , size factor and value factor of 1 ,3 and -2.

**Extension of Fama French Model (Carhart Model)**

There have been other interesting characteristic-based approaches to estimating a multifactor model of risk and return. Three of those approaches are described here. First, Carhart directly extends the Fama-French three-factor model by including a fourth common risk factor that accounts for the tendency for firms with positive (negative) past returns to produce positive (negative) future returns. He calls this additional risk dimension a *momentum factor* and estimates it by taking the average return to a set of stocks with the best performance over the prior year minus the average return to stocks with the worst returns. In this fashion, Carhart defines the momentum factor—which he labels *PR*1*YR*—in a fashion similar to *SMB* and *HML.* Formally, the model he proposes is:

**Ri = Rf + β1 mkt factorRMRF + β2 size factorSMB + β3 value factorHML + β4 momentum factor*PR*1*YR***

He demonstrates that the typical factor sensitivity (i.e., factor beta) for the momentum variable

is positive and its inclusion into the Fama-French model increases explanatory power by as much

as 15 percent.

**Extension of Fama French Model (Pastor and Stambough,2003)**

Another extension of Fama French work is the Pastor and Stambough model or PSM . PSM claims that investors require additional risk premium for holding illiquid share and adds fourth factor to FFM . Liquidity premium estimate is found by taking the difference between the return on illiquid shares and return on the liquid shares and takes the following form.

**Ri = Rf + β1 mkt factorRMRF + β2 size factorSMB + β3 value factorHML + β4 liquidity factor*LIQ***

**Build Up Model Estimate of Estimating Required Rate of Return**

Another widely used tool of estimating the required rate of return especially for private and closely held companies is the Buildup method. The method sums up risk free rate plus one or more risk premium to estimate the return

Ri = Risk free rate + one or more risk premium

Suppose we want to cost of equity or required return on equity using build up method.

Risk free rate is 6%. The market risk premium is 7%. The average systematic risk or beta is 1 for a publicly traded company. Now suppose we want to estimate the required return on equity of a private company which is very small in size ,

As company is not publicly traded, so its illiquid share, so investor will demand liquidity premium. Similarly due to small size, investor will require size premium. So we can add this risk premium to the required return of a average systematic risky publicly traded company to find the the required return of that private , small size company as follows

R­I =  6% + 7%\*1 + liquidly premium + Size premium

If Liquidity premium is = 2% and size premium is 3%, so the required return is

 Ri = 6% +7%\*1 + 2% + 3%

 = 18%.

**Bond Yield Plus Risk Premium (BYPRP)**

For companies with publicly traded debt , BYPRP approach provides a quick estimate of the cost of equity.

The estimate is

BYPRP = YTM on company’s long term debt + Risk Premium

YTM on company’s long term debt covers both

1. Real interest rate and premium for expected inflation
2. A default risk premium for bond obligation

The risk premium in BYPRP estimates covers the additional default risk arising from equity issue.

Suppose ACI has corporate bond listed with DSE having a yield of 12.5%. if Risk premium for equity issue is 4% then

BYPRP estimate of cost of equity = 12.5% + 4%

 = 16.5%

**Calculating Weighted Average cost of equity**

When calculating WACC, we need to have weight of different sources of capital such as debt, preference share and common stock. This weight may be based on market value weight, book value weight, and the target weight of each sources of capital in the capital structure.

Book value weight does do reflect the most recent market condition as most of assets are reported on Balance sheet using historical cost basis. So Book value weight is least preferred. Market value weight has, though reflects the most recent market condition, but is affected by the short term market price variation. Plus the current capital structure may not reflect what a company target. For example company may have debt to equity ratio 30% but company may find that if it can have capital structure at 50% debt and 50% equity , its WACC will be lowest, which is its target capital structure . So when estimating WACC, weight preference should be **target weight first, then market value weight, then if none of two is found, Book value weight .**