**Arbitrage Pricing Theory**

Though CAPM claims that required return of an asset is a linear function of its systematic risk and the only risk factor in required return estimation is market factor but many of the empirical studies point out some serious deficiencies in the model as an explanation of the link between risk and return. There was mixed support for a positive linear relationship between rates of return and systematic risk for portfolios of stock, with some recent evidence indicating the need to consider additional risk variables or a need for different risk proxies.

One especially compelling challenge to the efficacy of the CAPM was the set of results suggesting that it is possible to use knowledge of certain firm or security characteristics to develop profitable trading strategies, even after adjusting for investment risk as measured by beta. Typical of this work were the findings of Banz, who showed that portfolios of stocks with low market capitalizations (i.e., “small” stocks) outperformed “large” stock portfolios on a risk-adjusted basis, and Basu, who documented that stocks with low price-earnings (P-E) ratios similarly outperformed high P-E stocks. More recent work by Fama and French also demonstrates that “value” stocks (i.e., those with high book value-to-market price ratios) tend to produce larger risk-adjusted returns than “growth” stocks (i.e., those with low book-to-market ratios) and small cap company tends to produce higher return than big cap company on risk adjusted basis.

Given the limitations of CAPM, the investment and academic community searched for an alternative asset pricing theory to the CAPM that was reasonably intuitive, required only limited assumptions, and allowed for multiple dimensions of investment risk. The result was the arbitrage pricing theory (APT), which was developed by Ross in 1976 has three major assumptions:

1. Capital markets are perfectly competitive.

2. Investors always prefer more wealth to less wealth with certainty.

3. The stochastic process generating asset returns can be expressed as a linear function of a

set of K risk factors (or indexes).

APT says that there are many risk factors that drive the return of stocks in contrast to single market factor in CAPM. The theory assumes that the stochastic process generating asset returns can be represented as a *K* factor model of the form:

***E*(*Ri*) λ0 λ1*bi*1 λ2*bi*2 . . . λ*kbik…………………………………………………………….* (APT)**

Where:

λ0 = the expected return on an asset with zero systematic risk expected return on asset i when all the risk factor premium is zero

λ*j* = the risk premium/ factor premium related to the *j*th common risk factor

*bij* =factor sensibility ; that is, how responsive asset *i* is to the *j*th common factor. (These are called factor betas or factor loadings.)

Two terms require elaboration: λ*j* and *bij*. As indicated, λ terms are the multiple risk factors expected to have an impact on the returns of *all* assets. Examples of these factors might include inflation, growth in gross domestic product (GDP), major political upheavals, or changes in interest rates. The APT contends that there are many such factors that affect returns, in contrast to the CAPM, where the only relevant risk to measure is the covariance of the asset with the market portfolio (i.e., the asset’s beta). Given these common factors, the *bij* terms determine how each asset reacts to the *j*th particular common factor. To extend the earlier intuition, although all assets may be affected by growth in GDP, the impact (i.e., reaction) to a factor will differ. For example, stocks of cyclical firms will have larger *bij* terms for the “growth in GDP” factor than will noncyclical firms, such as grocery store chains. Likewise, you will hear discussions about interest-sensitive stocks. All stocks are affected by changes in interest rates; however, some experience larger impacts. For example, an interest-sensitive stock would have a *bj* interest of 2.0 or more, whereas a stock that is relatively insensitive to interest rates would have a *bj* of 0.5. Other examples of common factors include changes in unemployment rates, exchange rates, and yield curve shifts. It is important to note, however, that when we apply the theory**, *the factors are not identified.***

**Illustration:** assume that there are two common risk factors: one related to unexpected changes in the level of inflation and another related to unanticipated changes in the real level of GDP. If we further assume that the risk premium related to GDP sensitivity is 0.03 and a stock that is sensitive to GDP has a bj (where j represents the GDP factor) of 1.5, this means that this factor would cause the stock’s expected return to increase by 4.5 percent (= 1.5 × 0.03). To develop this notion further, consider the following example of two stocks and a two factor model. First, consider these risk factor definitions and sensitivities:

λ 1= unanticipated changes in the rate of inflation. The risk premium related to this factor is 2 percent for every 1 percent change in the rate (k1 = 0.02)

λ 2 = unexpected changes in the growth rate of real GDP. The average risk premium related to this factor is 3 percent for every 1 percent change in the rate of growth (k2 = 0.03)

λ 0 = the rate of return on a zero-systematic risk asset (i.e., zero beta) is 4 percent (k0 = 0.04) Assume also that there are two assets (x and y) that have the following response coefficients to these common risk factors:

bx1 = the response of asset x to changes in the inflation factor is 0.50 (bx1 = 0.50)

bx2 = the response of asset x to changes in the GDP factor is 1.50 (bx2 = 1.50)

by1 = the response of asset y to changes in the inflation factor is 2.00 (by1 = 2.00)

by2 = the response of asset y to changes in the GDP factor is 1.75 (by2 = 1.75)

These factor sensitivities can be interpreted in much the same way as beta in the CAPM; that is,the higher the level of bij, the greater the sensitivity of asset i to changes in the jth risk factor.Thus, the response coefficients listed indicate that if these are the major factors influencing asset returns, asset y is a higher risk asset than asset x, and, therefore, its expected return should be greater. The overall expected return equation will be:

**E (Ri) = λ0 + λ1bi1 + λ2bi2**

 **= 0.04 + (0.02)bi1 + (0.03)bi2**

**Therefore, for assets x and y:**

**E (Rx) = 0.04 + (0.02)(0.50) + (0.03)(1.50)**

 **= 0.0950**

 **= 9.50%**

**And**

**E(Ry) = 0.04 + (0.02)(2.00) + (0.03)(1.75)**

 **= 0.1325**

 **= 13.25%**